UNIVERSITÉ CÔTE D'AZUR | GRADUATE SCHOOL FORMAL, PHYSICAL AND ENGINEERING SCIENCES

1) Field of study : PDE analysis

2) Internship topic : KINETIC EQUATIONS AND HYPOCOERCIVITY METHODS

3) Description :

The central motivation of this internship is the study of partial differential equations derived from the kinetic theory of gases. We thus consider a density of particles $f = f(t,x,v) \geq 0$ where $t \in \mathbf{R}^+$ is the time, $x \in \Omega \subset \mathbf{R}^d$ is the position and $v \in \mathbf{R}^d$ is the velocity. This density satisfies an equation of the type :

$$\partial_t f + \mathcal{T} f = \mathcal{C} f$$

where $\mathcal{T}$ is a transport operator and $\mathcal{C}$ a (linear) collision operator, which acts only on the velocity variable. It is also assumed that the collision operator enjoys dissipativity properties (in velocity). The challenge of hypocoercivity theory is to understand the interaction between the transport operator and the collision operator, and to determine whether, in certain cases, the mixing of these two operators can lead to global dissipativity. Typically, the question is if one can obtain an exponential return to global equilibrium of the solutions of the equation when the collision operator is coercive. This is a topic that has been widely studied over the last few years for several kinetic equations and in several functional frameworks [11, 6, 9, 8, 10].

The aim of this internship is to discover a new method of hypocoercivity which is based on a trajectorial approach and has very recently been introduced in [5]. In particular, it allows to handle the case of bounded domains with very general boundary conditions, as well as the case where a spatial weight in front of the collision operator can vanish over part of the spatial domain.

- As a first step, we'll focus on gaining a good understanding of the article and rewrite the proof in a simplified, non degenerated framework and in the case of the torus. Several questions can be explored after that.
- Let us first mention that several works have extended the notion of hypocoercivity to a discrete framework [3, 7, 2]. In this context, a numerical scheme that preserves the continuous qualitative and quantitative properties of the equation is developed, enabling the development and study of a robust numerical method for the numerical approximation of the kinetic equation. The development of such a scheme for the case of the theoretical result under consideration will then be considered.
- Another possibility is to generalize the above result to a collision operator involving fractional diffusion, as other hypocoercivity methods have already been extended to this framework [4, 1].

References

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FORMAL, PHYSICAL AND ENGINEERING SCIENCES

4) Internship level :	Master 1 or Master 2
5) Requirements :	Basic background in PDE
6) Duration :	4 to 6 months
7) Period :	January to July 2024
8) Laboratory :	LJAD
9) Contact :	Thomas Rey, thomas.rey@univ-cotedazur.fr, Professeur des Universités, Laboratoire Jean Alexandre Dieudonné (LJAD/CNRS UMR7351), Université Côte d'Azur, 28 avenue Valrose, 06108 Nice Cedex 2, France Isabelle Tristani, isabelle.tristani@univ-cotedazur.fr Laboratoire Jean Alexandre Dieudonne (LJAD/CNRS UMR7351), Université Côte d'Azur, 28 avenue Valrose 06108 Nice Cedex 2, France

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