UNIVERSITÉ CÔTE D'AZUR | GRADUATE SCHOOL FORMAL, PHYSICAL AND ENGINEERING SCIENCES

1) Field of study : Fluid mechanics, PDE

2) Internship topic : Hamiltonian super-regularization of the inviscid Burgers Equation

3) Description :

The inviscid Burgers equation (or Hopf equation) $u_t + uu_x = 0$ is an equation with partial derivatives, nonlinear, conservative and hyperbolic. This equation appears as a simplified model in many physical, mechanical, chemical, etc. situa- tions. This equation develops singularities in finite time which can be problematic for some applications. Also, certain modifications of this equation have been pro- posed in order to avoid the appearance of these singularities; for example by adding viscosity or dispersion. However, these modifications are not sufficiently effective and/or destroy the symmetries of the equation (Hamiltonian structure).

A Hamiltonian regularization of the Hopf equation was recently proposed [1] and comes to be rigorously justified [2]. The advantage of this regularization is to be non-diffusive, non-dispersive and to preserve the fundamental properties of the original equation. Discontinuous solutions of the Hopf equation correspond to con- tinuous solutions of the regularized equation, but with discountinuous derivatives. It is therefore interesting to investigate whether there exists stronger Hamiltonian regularizations leading to more regular solutions, while preserving the structures of the original equation.

We propose to explore the possibility of stronger regularizations of the Hopf equation. These Hamiltonian super-regularizations will be introduced via varia- tional principles. The equations thus obtained will then be studied mathematically and/or numerically. This is a quite exploratory work, which can be approached in various ways and, therefore, leaving much freedom for the students to choose their orientation according to their preferences. This subject could lead to a thesis.

References:

[1] Guelmame, B., Junca, S., Clamond, D. and Pego, R. 2022. Global weak solutions of a Ha- miltonian regularised Burgers equation. J. Dyn. Diff. Eq. (in press).

[2] Guelmame, B. 2023. On a Hamiltonian regularization of scalar conservation laws. Disc. Cont. Dynam. Syst. (in press).

4) Internship level : Master 2

UNIVERSITÉ CÔTE D'AZUR EUR SPECTRUM

5) Requirements :	Basic background in PDE
6) Duration :	3 to 6 months
7) Period :	February to July 2024
8) Laboratory :	LJAD
9) Contact :	Didier Clamond, Didier.CLAMOND@univ-cotedazur.fr, Professeur des Universités, Laboratoire Jean Alexandre Dieudonne (LJAD/CNRS UMR7351) Université Côte d'Azur, 28 avenue Valrose, 06108 Nice Cedex 2, France. Stéphane Junca, Stephane.JUNCA@univ-cotedazur.fr, Maître de conférences, Laboratoire Jean Alexandre Dieudonne (LJAD/CNRS UMR7351) Université Côte d'Azur, 28 avenue Valrose, 06108 Nice Cedex 2, France.