UNIVERSITÉ CÔTE D'AZUR | GRADUATE SCHOOL **FORMAL, PHYSICAL AND ENGINEERING SCIENCES**

1) Field of study : Probability, differential equations

2) Internship topic : MULTIPLICATIVE REGULARIZATION BY NOISE

3) Description :

It is well-known that without rather standard Lipschitz-type conditions on b, uniqueness of solutions for differential equations of the form fails (even existence may pose a challenge).

$$X'(t) = b(X(t)), \quad X(0) \in \mathbb{R}^d.$$

A typical example is the celebrated Peano example, $d = 1$, $b(x) = 2\sqrt{|x|}$ and $X(0) = 0$.

Nevertheless, it is quite well-known that an additive perturbation via a Brownian motion of the equation may restore uniqueness when b is only Hölder continuous or even worse (see for example [4, 7]). Of course, one has to focus on equations of the form :

$$\mathrm{d}X_t = b(X_t)\,\mathrm{d}t + \mathrm{d}W_t, \quad X_0 \in \mathbb{R}^d,$$

and has to make sense of the solutions in this stochastic context. Such a phenomenon is called *regularization by noise*. Note that in [4] an almost sure result of regularization by noise is proved (which can be read as "for almost all Brownian trajectories uniqueness holds").

Since then, almost sure results of regularization by noise have attracted lots of attention (see [1, 6]), allowing to consider wider class of noise and of vector field b. In two recent articles, Catellier and Duboscq [2] and Dareiotis and Gerescér [3] have consider the "multiplicative" case in the context of rough-paths theory [5]. Using different techniques, they proved that regularization by noise phenomenon happens in the context of random Rough Differential Equations of the form

$$\mathrm{d}X_t = b(X_t)\,\mathrm{d}t + \sigma(X_t)\,\mathrm{d}\mathbf{B}_t, \quad X_0 \in \mathbb{R}^d.$$

Note that the use of Malliavin calculus [9] in [2] allows to consider general Gaussian Rough paths at a price of the generality on the vector fields b, whereas the use of stochastic sewing lemma techniques [8] allows the authors of [3] to have better generality on the vector filed b, but with less generality on the rough-paht $\mathbf{B}$.

In this internship we propose to address the following question : is it possible to match the techniques of [2, 3] together ? To answer such a question, we plan to focus ourselves first on standard Brownian motion, where both techniques (Malliavin calculus and stochastic sewing lemma) may be implemented in a rather simple way.

RÉFÉRENCES

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[4] Alexander M. Davie. Uniqueness of Solutions of Stochastic Differential Equations. *International Mathematics Research Notices*, 2007, January 2007.

[5] Peter Friz and Martin Hairer. *A Course on Rough Paths : With an Introduction to Regularity Structures.* Springer International Publishing AG, Place of publication not identified, October 2014.

[6] Lucio Galeati and Massimiliano Gubinelli. Noiseless regularisation by noise. *Revista Matemática Iberoamericana*, July 2021.

[7] Nicolai V. Krylov and Michael Roeckner. Strong solutions of stochastic equations with singular time dependent drift. *Probability theory and related fields*, 131(2) :154–196, 2005.

[8] Khoa Lê. A stochastic sewing lemma and applications. *Electronic Journal of Probability*, 25(none) :1–55, January 2020.

[9] David Nualart. *The Malliavin Calculus and Related Topics*, volume 1995. Springer, 2006.

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4) Internship level :	Master 2
5) Requirements :	Basic backgound in PDE and probability
6) Duration :	3 to 6 months
7) Period :	From February to July 2024
8) Laboratory :	LJAD
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