1) **Field of study**: Probability, differential equations

2) **Internship topic**: MULTIPLICATIVE REGULARIZATION BY NOISE

3) **Description**:

   It is well-known that without rather standard Lipschitz-type conditions on $b$, uniqueness of solutions for differential equations of the form fails (even existence may pose a challenge).

   \[ X'(t) = b(X(t)), \quad X(0) \in \mathbb{R}^d. \]

   A typical example is the celebrated Peano example, $d = 1$, $b(x) = 2\sqrt{|x|}$ and $X(0) = 0$.

   Nevertheless, it is quite well-known that an additive perturbation via a Brownian motion of the equation may restore uniqueness when $b$ is only Hölder continuous even worse (see for example [4, 7]). Of course, one has to focus on equations of the form:

   \[ dX_t = b(x_t) \, dt + dW_t, \quad X_0 \in \mathbb{R}^d, \]

   and has to make sense of the solutions in this stochastic context. Such a phenomenon is called *regularization by noise*. Note that in [4] an almost sure result of regularization by noise is proved (which can be read as “for almost all Brownian trajectories uniqueness holds”).

   Since then, almost sure results of regularization by noise have attracted lots of attention (see [1, 6]), allowing to consider wider class of noise and of vector field $b$. In two recent articles, Catellier and Delbaen [3] and Dereich and Gerencsér [4] have considered the “multiplicative” case in the context of rough-paths theory [5]. Using different techniques, they proved that regularization by noise phenomenon happens in the context of random Rough Differential Equations of the form

   \[ dX_t = b(X_t) \, dt + \sigma(X_t) \, dB_t, \quad X_0 \in \mathbb{R}^d. \]

   Note that the use of Malliavin calculus [9] in [2] allows to consider general Gaussian Rough paths at a price of the generality on the vector fields $b$, whereas the use of stochastic sewing lemma techniques [9] allows the authors of [3] to have better generality on the vector filed $b$, but with less generality on the rough-path $B$.

   In this internship we propose to address the following question: is it possible to match the techniques of [2, 3] together? To answer such a question, we plan to focus ourselves first on standard Brownian motion, where both techniques (Malliavin calculus and stochastic sewing lemma) may be implemented in a rather simple way.

**Références**


| 4) Internship level          | Master 2                     |
| 5) Requirements              | Basic background in PDE and probability |
| 6) Duration                  | 3 to 6 months                |
| 7) Period                    | From February to July 2024   |
| 8) Laboratory                | LJAD                         |
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