UNIVERSITÉ CÔTE D'AZUR

GRADUATE SCHOOL
FORMAL, PHYSICAL AND ENGINEERING SCIENCES

1) Field of study : Algebra, quantum Physics

2) Internship topic : Ideals of Grassmann algebra

3) Description :

The Grassmann algebra can be introduced with various motivations:

- to associate to n vectors of a vector space a number generalizing the notion of Euclidean volume, (zero when a certain linear dependence relation exists between the vectors).
- to generalize the algebras of polynomials in the case where the indeterminates anticommute.
- to decompose the elements of a tensor algebra according to their behavior under the action of the group of permutations.

Concretely, the exterior algebra of a vector space H of dimension m is generated by the base field, K, and the elements of the form $\varphi_1 \wedge \varphi 2 \wedge \quad \wedge \varphi p$, with $0 < p \leq m$,
and, for any i, $\varphi_{\in} H$, called "p-decomposable vectors"; here, the "external product"
$\wedge$ has the following property

$$\varphi_1 \wedge \varphi_2 \wedge \ldots \wedge \varphi_p = E_\sigma \varphi_{\sigma(1)} \wedge \varphi_{\sigma(2)} \wedge \ldots\ldots\ldots \wedge \varphi_{\sigma(p)}$$

for any permutation σ of the indices $\{1, \quad , p\}$, with signature E_σ.

Old works on this algebra, written in a style which is no longer current today, are often forgotten. However, they can prove to be very useful for the quantum theory of fermionic systems, such as the electrons of an atom or molecule or the nucleons of a kernel, whose wave functions are elements of the exterior algebra of a Hilbert space (see P. Cassam-Chena, F. Patras, Journal of Mathematical Physics, 44, p.4884-4906, 2003; Physics Letters, A 326, p. 297-306, 2004). The internship will consist of translating into modern language the following articles taken from the volume XXIV. Algebra and Number Theory, Chap. VI External Algebra, A. Chatelet ed., CNRS, Paris, 1950: On certain ideals of external algebra of degree $2n$, $n > 1$, by T. H. Lepage, p.181-186. Exterior algebra and the theory of elementary divisors, by G. Papy, p.187-191. We will then consider the potential applications of the results to quantum physics, in particular their usefulness in extending the recent proposal of T. M. Henderson and G. E. Scuseria in Geminal- based configuration interaction, J. Chem. Phys. 151, 051101 (2019), for the construction of general configuration interaction methods. This work is therefore likely to lead to an interdisciplinary Mathematics-Quantum Physics thesis.

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4) Internship level :	Master 2
5) Requirements :	Basic background in algebra
6) Duration :	3 to 6 months
7) Period :	February to July 2024
8) Laboratory :	LJAD
9) Contact :	Patrick Cassam-Chenaï, Patrick.CASSAM-CHENAI@univ-cotedazur.fr, chargé de recherche, Laboratoire Jean Alexandre Dieudonne (LJAD/CNRS UMR7351) – Université Côte d'Azur, 28, avenue Valrose, 06108 Nice Cedex 2, France.

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