# Workshop ANR ChaMaNe
# Challenges in Mathematical Neuroscience

## February 20 – 23, 2023

Salle de conférence, Dieudonné building, Université Côte d'Azur,
Parc Valrose, 06000 Nice, France

Tuesday February 20th

10h-10h30 Welcome - coffee

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|---|---|
| 10h30-11h30 | [Bio] Michele Bertacchi (iBV, Nice)<br>*Cortical neurogenesis: experimental observation and possible mathematical modeling* |
| 11h30-12h30 | [Bio] Pierre-Yves Jacob (LNC, Marseille)<br>*New properties of space coding neurons in complex environment.* |
| 12h30-13h30 | Lunch break (Rest. Admin. des impôts, rue Cadéi Nice) |
| 13h30- 15h30 | Long break to work (and/or collaborate on the biological open questions presented on Tuesday morning) |
| 15h30-16h30 | Delphine Salort (LCQB, Sorbonne Université)<br>TBA |
| 16h30-17h30 | Eric Luçon (MAP5, Paris Cité)<br>*Hawkes processes and the Neural Field Equation on the real line* |

Wednesday February 21st

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| 9h30-10h30 | Olivier Faugeras (Inria, Université Côte d'Azur)<br>*A new twist on the large size limit behaviour of networks of Hopfield-like neurons* |

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| 10h30-11h15 | coffee break |
| 11h15-12h15 | Nicolas Torres (IMAG, Granada)<br>*Analysis of an elapsed time model with discrete and distributed delays. New insights and theory.* |
| 12h15-13h30 | Lunch break (Rest. Admin. des impôts, rue Cadéi Nice) |
| 13h30- 15h30 | Long break to work (and/or collaborate on the biological open questions presented on Tuesday morning) |

Possible change of room exceptionally only for Wednesday afternoon :
we will warn you in due time of the room

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| 15h30-16h30 | Zoé Agathe-Nerine (MAP5, Paris Cité)<br>*Long-term stability of interacting Hawkes processes* |
| 16h30-17h30 | Grégory Faye (IMT, Toulouse)<br>TBA |

<u>Thursday February 22nd</u>

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|---|---|
| 9h30-10h30 | Susanne Solem (Dpt Math., Norwegian University of Life Science )<br><br>*A stochastic neural field in the form of a PDE with a tricky boundary condition* |
| 10h30-11h15 | coffee-break |
| 11h15-12h15 | Emre Baspinar (Inria, Université Côte d'Azur)<br>*A biologically plausible decision-making model based on interacting cortical columns* |
| 12h15-13h30 | Lunch break (Rest. Admin. des impôts, rue Cadéi Nice) |

13h30- 15h30 Long break to work (and/or collaborate on the biological open questions presented on Tuesday morning)

15h30-16h30 Anna Melnykova (Avignon)
*Granger Causal Inference in Multivariate Hawkes Processes by Minimum Message Length*

16h30-17h30 Francis Filbet (IMT, Toulouse)
TBA

19h30 Fancy dinner at restaurant

Friday February 23rd

9h00-9h45 Sophie Jaffard (LJAD, Nice)

*Provable local learning rule for a Hawkes network*

9h45-10h30 Qingyou He (LCQB, Sorbonne Université)

TBA

10h30-11h15 coffee break

11h15-12h15 Samuel Deslauriers-Gauthier (Inria, Université Côte d'Azur)

TBA

12h15-13h30 Lunch break (Rest. Admin. des impôts, rue Cadéi Nice)

THE END

# ABSTRACTS

## [Bio] Michele Bertacchi (iBV, Nice)

*Cortical neurogenesis: experimental observation and possible mathematical modeling*

The cerebral cortex is a highly complex structure responsible for various cognitive functions, and understanding how corticogenesis progresses during embryonic development is crucial for unraveling brain development and function. At early phases of neural development, self-renewing symmetric divisions increase the number of progenitor cells, while later steps of neurogenesis are characterized by asymmetric differentiative divisions that generate neurons. These two opposite processes need to be tightly controlled in time and space. While the general structure of progenitor and neuronal layers is maintained in different regions along the antero-posterior axis of the neocortex, the neuronal output (*i.e.* the number of neurons locally produced) can greatly vary, as the progenitors in each region of the cortex can behave differently.
Some determining factors that need to be considered for a mathematical model are the size and type of the founder population (apical vs basal), the duration of cortical neurogenesis, the proportion of different progenitor types, their spatial arrangement, and the fine-tuned balance between self-renewing and differentiative divisions (symmetric vs asymmetric). Our team is in particular interested by the role of the expression of a particular gene: the Nr2f1. We previously unraveled the role of the Nr2f1 gene to fine-tune the cell cycle dynamics and to control the neuronal output in the posterior-most cortex, but its role in other areas is still unknown. On the experimental side, Nr2f1 mutant mice can be used to take into account this regionalized aspect of neurogenesis, by measuring distinct rates of corticogenesis along the antero-posterior axis.
Existing models or simulations are based on ODE or agent-based simulation but do not take into account the spatial aspect, that is the regionalized aspect of corticogenesis. Our goal is to collaborate to integrate mathematical modelling and experimental observations to highlight the different strategies and factors that can explain regional diversity along the antero-posterior axis of the mammalian neocortex in terms of number of locally produced neurons.

## [Bio] Pierre-Yves Jacob (LNC, Marseille)

*New properties of space coding neurons in complex environment.*

Spatial navigation relies on a mental representation of space supported by the activity of space-coding neurons including place cells (PC), head direction cells (HDC) and grid cells (GC). The activity of PC, GD and HDC has been mostly exclusively described in animals

navigating in simple environments, in which they are trained to move to collect rewards randomly scattered on the floor. However, in everyday life, rodents or humans constantly move between connected places or rooms in which they have to find their way and most importantly to reach for goals. Since most natural environments are composed of distinct connected spaces that we explore for specific purposes, my research focuses on how spatial neurons integrate two relevant features of space: a spatial goal and the connectivity between different rooms.
My talk will be mainly based on two recent unpublished experiments. In the first study, I will show that the retrosplenial cortex contains two populations of head direction cells that may allow navigating between connected environments. In a second study, I will show that entorhinal grid cells lose their striking regular spatial pattern when an animal has to reach for a spatial goal in an environment.
By discovering new properties of these spatial neurons in complex spaces, our team tries to understand the neurobiological bases of the cognitive map in more ecologically relevant environments than the ones traditionally used in laboratory settings.

## Zoé Agathe-Nerine (MAP5, Paris Cité)

*Long-term stability of interacting Hawkes processes*

We consider a multivariate Hawkes process modeling the activity of $N$ interacting neurons, regularly positioned on a bounded space. We study the model and its large population limit, that can be described by the neural field equation (NFE). We study the long-time stability of the synaptic current of the population, as $N \to \infty$, in case the synaptic memory kernel is exponential, up to time horizons that are polynomial in $N$ in two different cases: one where there is a unique stationary solution to the NFE, and one where the NFE admits a locally stable manifold of stationary solutions. For the latter case, we show in particular that the phase of the voltage along this manifold converges towards a Brownian motion on a time scale of order $N$.

## Emre Baspinar (Inria, Université Côte d'Azur)

*A biologically plausible decision-making model based on interacting cortical columns*

The Adaptive Exponential (AdEx) mean-field framework [1] describes the averaged neuronal population dynamics modeled by the AdEx network. In the case of cerebral cortex, AdEx networks are used to model two cell types: Regular Spiking (RS) neurons, displaying spike-frequency adaptation as observed in excitatory pyrami- dal neurons, and Fast Spiking (FS) neurons, with no adaptation, as observed in inhibitory interneurons. AdEx networks are high dimensional, complex and difficult to analyze. AdEx mean-field models are low dimensional, simpler and easier to analyze compared to networks, yet they approximate closely the network dynamics, motivating our choice of model. Here, we extend the AdEx mean-field framework to model two networks of excitatory-inhibitory neurons, representing two different cortical columns [2, 3] . The model is biophysically plausible since it is based on intercolumnar excitation modeling the long range connections and intracolumnar excitation-inhibition modeling the short range connections. This connection scheme introduces bicolumnar competition, sufficient for choosing between two alternatives. Each column represents a pool of neurons voting for one of the two alternatives indicated by two stimuli presented on a monitor. We endow the model with a reward- driven learning mechanism which captures the optimal strategy maximizing the cumulative reward, as well as to model the exploratory behavior of the participant. The model has been tested on behavioral results obtained from human and macaque experiments and it provides promising performance. This model contributes to a biophysical ground for simpler phenomenological models proposed for similar decision-making tasks and it can be applied to neurophysiological data. Finally, it can be embedded in whole-brain simulators, such as The Virtual Brain (TVB), to study decision-making in terms of large scale brain dynamics [4]. This is a joint work with Gloria Cecchini, Michael DePass, Marta Andujar, Pierpaolo Pani, Stefano Ferraina, Rubén Moreno-Bote, Ignasi Cos and Alain Destexhe.

References

[1] Di Volo M., Romagnoni A., Capone C., Destexhe A. (2019) Biologically realistic mean-field models of conductance-based networks of spiking neurons with adaptation. Neural Computation 31(5):653-680.

[2] Baspinar E., Cecchini G., DePass M., Andujar M., Pani P., Ferraina S., Moreno-Bote R., Cos I., Destexhe A. (2023) A biologically plausible decision-making model based on interacting cortical columns. bioRxiv 530384. Available from https://doi.org/10.1101/2023.02.28.530384.

[3] Baspinar E., Cecchini G., Moreno-Bote R., Cos I., Destexhe A. (2022) Jupyter notebook of a biophysically plausible decision-making model based on interacting cortical columns (v1.0.0) [Python source code]. Available from https://github.com/emrebasp/Jupyter-notebook- A-biophysically-plausible-decision-making-model-based-on-interacting-cortical-co.

[4] Turan A., Baspinar E., Destexhe A. (2023) A whole-brain model of auditory discrimination. bioRxiv 559095. Available from https://doi.org/10.1101/2023.09.23.559095.

## Olivier Faugeras (Inria, Université Côte d’Azur)

*A new twist on the large size limit behaviour of networks of Hopfield-like neurons*

We revisit the problem of characterising the thermodynamic limit of a fully connected network of Hopfield-like neurons. Our contributions are that we provide a) a complete description of the mean-field equations as a set of stochastic differential equations depending on a mean (m(t)) and covariance (K(t, s)) functions, b) a provably convergent method for estimating these functions, and c) numerical results of this estimation as well as examples of the resulting dynamics. The mathematical tools are the theory of Large Deviations, Itô stochastic calculus, and the theory of Volterra equations.
We have generalized and set on a firm mathematical basis the work of Sompolinsky et al., e.g. [2], Helias et al., e.g. [4]. We have also extended the work of Ben Arous and Guionnet, e.g. [1], and clarified and complemented the work of Faugeras et al., e.g. [3].
Joint work with Etienne Tanré.

References

[1]Gérard Ben Arous and Alice Guionnet. Large deviations for Langevin spinglass dynamics. Probability Theory and RelatedFields,102(4):455– 509, 1995.

[2] A. Crisanti and H. Sompolinsky. Path integral approach to random neural networks. Physical Review E, 98(6):062120, December 2018.

[3] Olivier Faugeras,Jonathan Touboul,and Bruno Cessac.A. constructive mean-field analysis of multipopulation neural networks with random synaptic weights and stochastic inputs. Frontiers in Computational Neuroscience, 3, 2009.

[4] Alexander van Meegen, Tobias Kühn, and Moritz Helias. Large-Deviation Approach to Random Recurrent Neuronal Networks: Parameter
Inference and Fluctuation-Induced Transitions. Physical Review Letters, 127(15):158302, October 2021.

## Sophie Jaffard (LJAD, Nice)

*Provable local learning rule for a Hawkes network*

Recordings of human brain suggest that concepts are represented through sparse set of neurons that fire when the concept is activated. Neuroscientists have identified local learning rules to adjust synaptic weights, but to our knowledge there is no mathematical proof that such local rules enable to learn. We use Hawkes networks as a model for a cognitive network that can learn categories by updating synaptic weights with a local learning rule. In this network, output nodes are post-synaptic neurons that produce spikes as a discrete-time Hawkes process, whose spiking probability is a function of the weighted sum of the activity of the pre-synaptic neurons at the previous time step. Kalikow decomposition allows us to interpret these synaptic weights in the previous sum as a probability distribution. This interpretation of the synaptic weights leads to the following local vision: for a postsynaptic neuron, its presynaptic neurons can be seen as so many experts and the distribution, given by the weights, can be related to an expert aggregation problem. This is why we use at this stage an expert aggregation algorithm to update the weights, which provides a local learning rule. The resulting algorithm is called HAN (Hawkes Aggregation of Neurons) and is general enough for any expert aggregation algorithm. We were able to prove mathematically that HAN is able to learn. To the best of our knowledge, the present work mathematically proves for the first time that such local learning rules make a very simple network learn.

## Eric Luçon (MAP5, Paris Cité)

*Hawkes processes and the Neural Field Equation on the real line*

This is an ongoing work with C. Poquet. The Neural Field Equation (NFE) was introduced by Wilson, Cowan and Amari to model mesoscopic activity of neural networks. It has particularly proven to exhibit nontrivial dynamical structures such as traveling waves.

Recently, Chevallier, Duarte, Löcherbach and Ost have given a microscopic interpretation of the NFE in terms of the limit of the potential of spatially organized Hawkes processes. Whereas the previous work essentially restricts to bounded spatial domains, the aim of the talk is to discuss a similar approximation on the real line. We will give arguments in favor of the stability of such particle system w.r.t. traveling waves, on a time scale of order the size of the population.

## Anna Melnykova (Avignon)

*Granger Causal Inference in Multivariate Hawkes Processes by Minimum Message Length*

Multivariate Hawkes processes (MHPs) are versatile probabilistic tools used to model various real-life phenomena: earthquakes, operations on stock markets, neuronal activity, virus propagation and many others. In this work, we focus on MHPs with exponential decay kernels and estimate connectivity graphs, which represent the Granger causal relations between their components. We approach this inference problem by proposing an optimization criterion and model selection algorithm based on the minimum message length (MML) principle. While most of the state-of-art methods using lasso-type penalization tend to overfitting in scenarios with short time horizons, the proposed MML-based method achieves high F1 scores in these settings. We conduct a numerical study comparing the proposed algorithm to other related classical and state-of-art methods, where we achieve the highest F1 scores in specific sparse graph settings. We illustrate the proposed method also on G7 sovereign bond data and obtain causal connections. It is a joint work with Irene Tubikanec (Klagenfurt University) and Katerina Hlavackova-Schindler (University of Vienna).

## Susanne Solem (Dpt Math., Norwegian University of Life Science)

*A stochastic neural field in the form of a PDE with a tricky boundary condition*

A Fokker--Planck-like partial differential equation was recently proposed to represent stochastic neural fields with the aim of better understanding the impact of noise on grid cells. This representation allows a study of noise-induced behaviors in a deterministic framework. However, the nonlinear and non-local nature of the PDE combined with a no-flux boundary condition prevents a direct application of methods used to study properties of more classical neural field models, such as pattern formation and stability of stationary states. In this talk I will present recent results for this model, and discuss some challenges related to it. The talk is based on works with Jose A. Carrillo, Helge Holden, and Pierre Roux.

## Nicolas Torres (IMAG, Granada)

*Analysis of an elapsed time model with discrete and distributed delays. New insights and theory.*

The elapsed time equation is an age-structured model that describes dynamics of interconnected spiking neurons through the elapsed time since the last discharge, leading to many interesting questions on the evolution of the system from a mathematical and biological point of view. In this talk, we first deal with the case when transmission after a spike is instantaneous and the case when there exists a distributed delay that depends on previous history of the system, which is a more realistic assumption. Then we study the well-posedness and the numerical analysis of these elapsed time models. For existence and uniqueness we improve the previous works by relaxing some hypothesis on the non-linearity, including the strongly excitatory case, while for the numerical analysis we prove that the approximation given by the explicit upwind scheme converges to the solution of the non-linear problem. We show some numerical simulations to compare the behavior of the system under different parameters, leading to solutions with different asymptotic profiles. Moreover, we present some new perspectives that are interesting to determine the asymptotic behavior of the system.